Indian J. Phys. **43**, 499—506 (1969)

Theoretical design of the experiment to study scattering of electrons and positrons by electrons

By M. R. Bhiday, V. M. Bhise and A. G. Shastri

G. S. Institute of Technology and Science, Indore.

(*Received* 24 *July* 1969)

Experiments to measure the electron-electron and positron-electron scattering cross-sections using a transverse magnetic field are in progress in this laboratory. The theoretical design study of these experiments is discussed in this paper. Monoenergetic beams of electrons or positrons are incident nearly normally to the scattering foil placed in the plane of the magnetic field. It is shown that the scattered and recoil particles due to electron-electron or positron-electron collisions with energy transfer q are focussed on two lines along the magnetic field. Relations between the x and z coordinates of the scattered particles, when focussed on the lines and the angles of scattering, are derived enabling the design and exact positioning of the scintillators in order to achieve a high collection efficiency and good resolution.

Introduction

A direct check on the Dirac theory of the electron is provided by the comparison of experimental results on electron-electron scattering as per the predictions of quantum-electrodynamics. When the incident electron has energies in the relativistic region, the problem becomes one of quantum-electrodynamics, an exact solution of which cannot be obtained directly but must be approached through an expansion in terms of the interaction constant $\frac{e^2}{hv}$.

The Moller formula (1932) for the relativistic electron-electron scattering cross-section may be written very conveniently in terms of the energy transfer occurring in the collision rather than in terms of the scattering angle. If the fractional kinetic energy transfer is denoted by q, the differential scattering cross-section becomes (Mott & Massey 1949)

$$\sigma(q)\,dq = 2\pi\left(\frac{e^2}{mv^2}\right)^2\left(\frac{1}{\gamma-1}\right)\left[X^2-3X+\left(\frac{\gamma+1}{\gamma}\right)^2(1+X)\right]dq$$

where,

$$X = [q\,(1-q)]^{-1}.$$

We have done experiments to verify the above formula by measuring the cross-section for electron-electron scattering accurately to 5%. Results of this accuracy have not been reported so far by earlier workers (Ashkin *et al* 1954). Movable pairs of scintillation counters have been used for the first time in our experiments, the results of which will be published separately. The purpose of this paper is to report on the theoretical design of the experiment to measure the electron-electron scattering cross-section, in which the line focussing property (Bhiday & Tripathi 1962), for the scattered electrons in a transverse magnetic field has been brought out clearly and used in the design (Bhatnagar *et al* 1965) of the scintillator shape.

Theoretical Design of the Experiment

In our experimental set up a beam of electrons, energy selected by a 60° sector magnet (Bhattacharya & Bhiday 1967), is directed on to a thin scattering foil with its plane parallel to a uniform magnetic field. Any electron-electron scatter in the foil gives rise to the scattered and the recoil electrons. These were detected in coincidence by two long scintillators placed in a plane parallel to the field and containing the foil. By measurements of the coincidence counting rate, the foil thickness and the incident beam intensity, the scattering cross-section was deduced. In order to measure the scattering occurring in a particular angular range, the two scintillators must be placed so that only for scatters occurring within this range can both scattered and recoil electrons be detected simultaneously. To position these scintillators correctly, it was necessary to know the relationship between the points where the scattered electrons cross the scintillator plane and the scattering angle (figure 1).

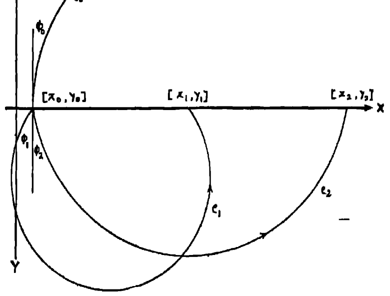

Figure 1. Projection on the XY plane.

This relationship can most conveniently be derived using a set of Cartesian axes defined so that the Z direction was that of the magnetic field and the XZ plane contained the scattering foil and the scintillator surfaces.

Figures 1 and 2 show the projections on to the XY and XZ planes, respectively, of the electron orbits occuring due to an electron-electron collision in the foil. The particle e_0 represents an electron of total energy γ_0 rest masses incident upon the foil at a point (x_0, z_0). After collision the scattered and recoil electrons e_1 and e_2 travel in helices in the uniform magnetic field and recross the XZ plane at (x_1, z_1) and (x_2, z_2) respectively.

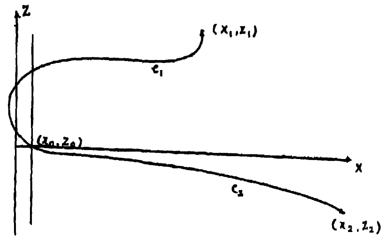

Figure 2. Projection on the XZ plane.

The electron paths at the point of collision are shown in more detail in figures 3 and 4.

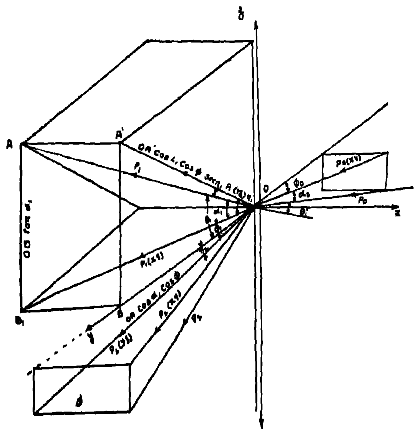

Figure 3. Electron trajectories at the point of collision

Let P_r, $_{r=0,1,2}$ represent the momenta in directions $(l_r, m_r, n_r)_{r=0,1,2}$ carried by e_r, $_{r=0,1,2}$ and let $P_{r(xy)r=0,1,2}$ denote projections of P_r in XY plane having direction cosines $(l_r', m_r', 0)$. Similarly $P_{r(yz)}$ denote the projection of P_r in YZ plane. If the pitch angle of helix in which the electron travels be denoted by α_r,

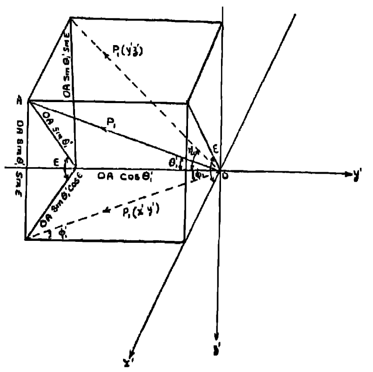

Figure 4. Electron trajectories at the point of collision

$$\alpha_r = \widehat{P_r, P_{r(xy)}} \text{ and } \theta_r = \widehat{P_r\, OY}, \phi_r = \widehat{P_{r(xy)}, OY}$$

$$\theta'_r = \widehat{P_0, P_r} = \text{Scattering angle and } \eta_r = \widehat{P_{r(yz)}, OY}.$$

From figure (3) we obtain

$$\cos \alpha_1 \cos \phi_1 = \cos \theta_1 \text{ or } l_1 l'_1 + m_1 m_1) \; m'_1 = m_1. \qquad .. (1)$$

and $\quad \tan \alpha_1 = \cos \phi_1 \tan \eta_1.$

As P_1, $P_1(YZ)$ and OX are coplanar, projecting OA along OA_1 gives .. (2)

$$\cos \alpha_1 \cos \phi_1 \sec \eta_1 = m_1 \sec \eta_1 = (1-l_1^2)^{\frac{1}{2}}, \qquad .. (3)$$

where $\quad l_1 = \cos \alpha_1 \sin \phi_1.$

From figure (1), it can be seen that,

$$x_1 - x_0 = 2\rho_1 m'_1 \qquad .. (4)$$

and

$$2\rho_1 = \beta_1 \gamma_1 (l_1 l'_1 + m_1 m'_1) \bar{M}, \qquad ...(5)$$

where

$$\bar{M} = \frac{2M_0 c}{eH}.$$

Herein ρ_1 is the radius of helix in which e_1 is travelling, H = magnetic field, β_1 = velocity of e_1 relative to that of light. Substituting for ρ_1 from (5) and using (1) we get,

$$x_1 - x_0 = \bar{M}\, \beta_1 \gamma_1 m_1 \qquad (6)$$

To obtain m_1 in terms of θ'_1—the scattering angle—consider a new set of orthogonal axes where OY' lies along P_0, OX' lies in the XY plane and OZ' in the direction perpendicular to OX' and OY'.
From figure 4,

$$\phi'_r = \widehat{P_r(x'y'), OY} \qquad r = 1, 2$$

and ϵ = angle between the planes of P_0P_1 and $X'Y'$.

Hence $\tan\phi'_1 = \tan\theta'_1 \cos\epsilon.$ ...(7)

Some direct relations are :

$$\left.\begin{aligned} &\cos\phi_0 = m'_0,\ \sin\phi_0 = l'_0\ ; \\ &\cos\alpha_0 = (l_0l'_0 + m_0m'_0) = N_1 \text{ say and} \\ &\sin^2\alpha_0 = n_0^2 + (l_0m'_0 - m_0l'_0)^2 = N_2^2 \text{ say.} \end{aligned}\right\} \quad ...(8)$$

$$\left.\begin{aligned} &\sin^2\theta'_1 = \Sigma(m_0n_1 - m_1n_0)^2 = \frac{2(1-q)}{2+q(\gamma_0-1)} = A^2 \text{ say,} \\ &\cos^2\theta'_1 = (\Sigma\, l_0l_1)^2 = \frac{q(\gamma_0+1)}{2+q(\gamma_0-1)} = B^2 \text{ say} \\ \text{and}\quad &\beta_1^2\gamma_1^2 = [2+q(\gamma_0-1)]\, q\,(\gamma_0-1) \end{aligned}\right\} \quad ...(9)$$

where q is the fractional kinetic energy tranfer occurring in the collision, since it is found more useful to put θ_1 in terms of q by equating moments and energies before and after the collision. As the direction cosines of OX' and OY' are m_0', l_0', 0 and l_0, m_0, n_0 respectively, therefore, the direction cosines of OZ' are

$$(l_0'n_0,\ -m_0'n_0,\ m_0m_0' - l_0l_0')/D \quad ...(10)$$

where $\quad D^2 = n_0^2 + (m_0m'_0 - l_0l'_0)^2.$

We wish to obtain m_1 in terms of ϵ and the direction cosines of P_0, P_1. Using (10) the direction cosines of OY with respect to new axes are :

$$-\,l'_0,\ N_1m'_0,\ N_2m'_0 \quad ...(11)$$

The direction cosiness of P_1 with respect to new axes are :

$$\left.\begin{aligned} &l_1m'_0 + m_1l'_0,\ \Sigma l'_0l_1,\ [l_1l'_0n_0 - m_1m'_0n_0 - n_1(m_0m_0' - l_0l'_0)]/D \\ &\text{and also} \\ &\sin\theta'_1\cos\epsilon\,, \quad \cos\theta'_1\,, \quad \sin\theta'_1\sin\epsilon \end{aligned}\right\} \quad ...(12)$$

From (12) using (9) we obtain,

$$\left.\begin{aligned} &\cos\epsilon = (l_1m'_0 + m_1l_0)/A \\ \text{and}\quad &\sin\epsilon = [l_1l'_0n_0 - m_1m'_0n_0 + n_1(m_0m'_0 - l_0l'_0)]/DA \end{aligned}\right\} \quad ...(13)$$

From (11) and (12) and substituting in terms of ϵ from (13) we have,

$$m_1 = A[m'_0N_2\sin\epsilon - l'_0\cos\epsilon\,] + B\,m'_0\,N_1 \quad ...(14)$$

Substituting in (6) for m_1 from (14) and for $\beta_1 \gamma_1$ from (9) we get,

$$x_1 - x_0 = \bar{M}\,(\gamma_0^2 - 1)^{\frac{1}{2}}(m_0\, N_1)\left[\, q + \frac{A}{N_1}\left(N_2 \sin \epsilon + \frac{l'_0}{m'_0} \cos \epsilon\right)\right] \qquad \ldots(15)$$

where $\bar{M}$, N_1, N_2 and A are defined in (5), (8) and (9). Similarly it can be shown that,

$$x_2 - x_0 = \bar{M}\,(\gamma_0^2 - 1)^{\frac{1}{2}}(m'_0\, N_1)\left[(1 - q) - \frac{A}{N_1}\left(N_2 \sin \epsilon - \frac{l'_0}{m'_0} \cos \epsilon\right)\right] \qquad \ldots(16)$$

From the equation to the helix, the z coordinate of the point where the scattered electron e_1 will cross the XZ plane is given by,

$$z_1 - z_0 = 2\rho_1\left(\frac{\pi}{2} - \phi_1\right) \tan \alpha_1 \qquad (17)$$

taking ϕ_1 as positive if the scatter is into the x positive quadrant and negative if the scatter is into the x negative quadrant.

Substituting for $\tan \alpha_1$ from (2) and using (6),

$$z_1 - z_0 = \left(\frac{\pi}{2} - \phi_1\right)\bar{M}\,\beta_1\gamma_1\,(1 - l_1^2 - m_1^2)^{\frac{1}{2}} \qquad \ldots(18)$$

Now we wish to obtain m_1 and ϕ_1 in terms of q and ϵ. For m_1 equating the values of $(x_1 - x_0)$ from (6) and (15) we get,

$$m_1\left[\frac{\beta_1\gamma_1}{m_0' N_1^2(\gamma_0^2 - 1)}\right] = q + \left[\frac{2q(1-q)}{\gamma_0 + 1}\right]^{\frac{1}{2}} \times \frac{1}{N_1}\left(N_2 \sin \epsilon - \frac{l'_0}{m'_0} \cos \epsilon\right) \qquad \ldots(19)$$

The coordinates of any point $P(-r \sin \phi_1, r \cos \phi_1, 0)$ on $P_1(xy)$ using (10) become

$$\left.\begin{array}{l} -r \sin \phi_1\, m'_0 + r \cos \phi_1 l'_0,\; -r \sin \phi_1 l_0 + r \cos \phi_1\, m_0, \\ \qquad (-r \sin \phi_1 l'_0 n_0 - r \cos \phi_1\, m'_0 n_0)/D \\ \text{and also,} \\ r \cos \alpha_0 \sin \phi'_1,\; -r \cos \alpha_0 \cos \phi'_1,\; r \sin \alpha_0 \end{array}\right\} \qquad \ldots(20)$$

as the $X'Y'$ plane is inclined at α_0 with XY plane. From (20) by equating x and y coordinates we get,

$$-\tan \phi'_1 = \frac{\sin \phi_0 - \cos \phi_0 \tan \phi_1}{m_0 - l_0 \tan \phi_1} \qquad \ldots(21)$$

From (21) and (7) we get,

$$\tan \phi_1 = \frac{l'_0 + \frac{A}{B} m_0 \cos \epsilon}{m'_0 + \frac{A}{B} l_0 \cos \epsilon} \qquad \text{...(22)}$$

Substituting in (18) the value of $\left(\frac{\pi}{2} - \phi_1\right)$ from (22),

$$(z_1 - z_0) = \cot^{-1}\left[\frac{l'_0 + \frac{A}{B} m_0 \cos \epsilon}{m'_0 + \frac{A}{B} l_0 \cos \epsilon}\right] \bar{M}\, \beta_1\gamma_1 \,(1 - l_1^2 - m_1^2)^{\frac{1}{2}} \quad \text{.. (23)}$$

where m_1 is given by (19).

Similarly it can be shown that $(z_2 - z_0)$ is obtained by replacing l_1 by l_2 and q by $(1 - q)$.

From equations (15) and (16) it can be seen that when α_0 and ϕ_0 are small, the x coordinates of the points where e_1 and e_2 cross the counter plane are independent of the azimuth of the plane of scattering. For a monoenergetic electron beam incident on a line $x = x_0$ on the scattering foil, all scattered and recoil electrons due to electron-electron collisions with energy transfer q will cross the counter plane as two lines given by

$$x_1 - x_0 = \bar{M}\, q(\gamma_0^2 - 1)^{\frac{1}{2}} \qquad \text{...(24)}$$

and

$$x_2 - x_0 = \bar{M}\,(1 - q)(\gamma_0^2 - 1)^{\frac{1}{2}} \qquad \text{.. (25)}$$

This is the line focussing property of the uniform transverse magnetic field used in the present experiment. This magnetic field was obtained between pole pieces 13 inches in diameter with a gap of 6 inches in which a scattering chamber containing the foil and the scintillators with the light guides could be inserted.

When $\alpha_0 \to 0$, $\phi_0 \to 0$, we have $l_0 = n_0 = 0$ and $m_0 = 1$ then (23) becames

$$z_1 - z_0 = \cot^{-1}\left(\frac{A}{B} \cos \epsilon\right) \bar{M}\, \beta_1\gamma_1\left(1 - l_1^2 - B^2\right)^{\frac{1}{2}} \qquad \text{...(26)}$$

The last bracket has a maximum value when $l_1 \to 0$ with $l_1 \to 0$, $\cos \epsilon \to 0$ and the first bracket reduces to $\frac{\pi}{2}$.

Then the maximum positive value of z shift is given by

$$(z_1 - z_0)_{\max} = \frac{\pi}{2} \bar{M}\, \beta_1\gamma_1(1 - B^2)^{\frac{1}{2}} \qquad \text{...(27)}$$

and the maximum negative value is given when $\epsilon \rightarrow \frac{3\pi}{2}$. For e_2 the position is reversed, the maximum positive and negative values of z_2 being when ϵ is near $\frac{3\pi}{2}$ and $\frac{\pi}{2}$ respectively. The mathematical treatment is true for positron-electron scattering also except that the positrons and electrons will be bent in opposite directions with the positron carrying away a kinetic energy denoted by factor $(1 - q)$.

The line focussing property mentioned above was used with advantage by making the detecting scintillators long in the Z direction and placing them on the focus lines corresponding to the scattering angle for which it is desired to make measurements. The scintillators were mounted on cylindrical light guides so that these could be moved along the X axis. In this way good collection efficiency was obtained without loss of angular resolution. Because of this efficiency, scattering measurements of good accuracy and resolution have been made using the relatively weak particle beams obtainable from radioactive sources. In addition to the economic considerations, the use of a radioactive source of particles has the advantage that, an apparatus of this type can be used either for electron-electron scattering measurements or for positron-electron scattering measurements. Both are under investigation in this laboratory using pairs of movable scintillation counters which enable scattering measurements to be done for continuously variable values of q, the kinetic energy transfer.

Thanks are due to Dr. S. M. Das Gupta, Director, G. S. Institute of Technology and Science.

References :

Ashkin, A., Page, L. A. & Woodward, W. M. 1954 *Phys. Rev*, **94**, 357.

Bhatnagar, K. G., Bhawalkar, D.V., Bhiday, M. R. 1965 *Ind. J. Pure & Appl. Phys.* **3**, 500.

Bhattacharya, P. K. & Bhiday. M. R. 1967 *Ind. J. Pure & Appl. Phys.* **5**, 386.

Bhiday, M. R. & Tripathi, R. N. 1962 *Nucl. Phys. Symp. Madras*, 193.

Moller C., 1932 *Ann. Physik.*, **14**, 531.

Mott, N. F. & Massey. H. S. W., 1949 *Theory of Atomic Collisions*, Oxford.